

DYNAMICAL ONE-DIMENSIONAL MATHEMATICAL MODEL OF A ROTARY KILN. PART 1: MAIN EQUATIONS

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Summary. General issues of mathematical description of rotary kiln are considered. Behavior of complex heat exchange, chemical reactions, changing of phase state and crystal structure of processed material is included. Dynamical one-dimensional mathematical model of rotary kiln is designed for definition of axial and time dependences of studied parameters.

Keywords: rotary kiln, mathematical model, conservation equations, mass transfer.

Conservation equations

Three phases can be included in the description of heat and mass exchange inside rotary kiln: processed material, dust and gas. Solid phase reactions have different kinetic coefficients for material and dust.

Continuity equations

Continuity equation for gas can be written in a form:

$$\frac{\partial}{\partial t} [\varepsilon_g \rho_g] + \frac{\partial}{\partial x} [\varepsilon_g \rho_g u_g] = \sum_{i=1}^{ng} \sum_{j=1}^{rs} (S_{sg(i,j)} + S_{dg(i,j)}), \quad (1)$$

supposing that vectors of velocity of dust and gas are equal, we can write for dust:

$$\frac{\partial}{\partial t} [\varepsilon_d \rho_d] + \frac{\partial}{\partial x} [\varepsilon_d \rho_d u_d] = P_1 - P_2 - \sum_{i=1}^{ng} \sum_{j=1}^{rs} S_{dg(i,j)}, \quad (2)$$

for material:

$$\frac{\partial}{\partial t} [\varepsilon_s \rho_s] + \frac{\partial}{\partial x} [\varepsilon_s \rho_s u_s] = - \left[P_1 - P_2 + \sum_{i=1}^{ng} \sum_{j=1}^{rs} S_{sg(i,j)} \right], \quad (3)$$

therewith (fig. 1):

$$\varepsilon_g + \varepsilon_d + \varepsilon_s = 1, \quad (4)$$

where $\partial/\partial t$ and $\partial/\partial x$ – time and kiln length derivatives consequently, s⁻¹ and m⁻¹; characteristics of gas, dust and material specified by sub-indexes g , d and s correspondently; ε – volumetric fraction; n_g and n_s – total number of chemical compounds in gas and material consequently; r_s – total number of interphase reactions and phase changes between material and gas with formation of gas substances; ρ – density, kg/m³; u – velocity, m/s; parameters of interface transfer specified by sub-indexes g

and d_g ; $\sum_{i=1}^{ng} \sum_{j=1}^{rs} S_{sg(i,j)}$ – summarized interface mass transfer

from material to gas as a result of r_s interface transfers material-gas and chemical reactions in material with formation of gas product with sub-index i , kg/(m³·s);

$\sum_{i=1}^{ng} \sum_{j=1}^{rs} S_{dg(i,j)}$ – the same for transfer dust-gas, kg/(m³·s);

P_1 and P_2 – intensity of raising and precipitation of dust, kg/(m³·s).

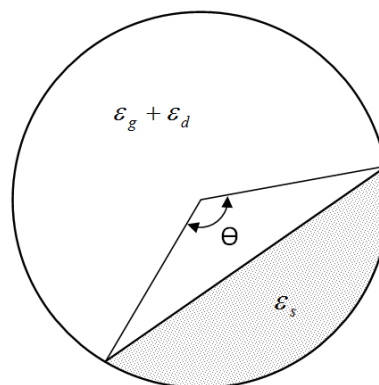


Fig. 1. Cross section of a rotary kiln.

Initial conditions for equations (1)-(3) (at $t = 0$):

$$\begin{aligned} \frac{d}{dt} \varepsilon_g \rho_g u_g(x, t) &= g_{g.init}(x) / F_f; \\ \frac{d}{dt} \varepsilon_d \rho_d u_g(x, t) &= g_{d.init}(x) / F_f; \\ \frac{d}{dt} \varepsilon_s \rho_s u_s(x, t) &= g_{s.init}(x) / F_f, \end{aligned} \quad (5)$$

where $g_{g.init}$, $g_{d.init}$ и $g_{s.init}$ – distribution functions for mass rates of gas, dust and material in axial direction of a kiln at the initial time point, kg/s; $F_f = 0,25\pi d_f^2$ – area of inside kiln cross-section as a function of axial coordinate, m².

Boundary conditions for equations (1)-(3) (at $x = x_{in}$):

$$\begin{aligned} \frac{d}{dx} \varepsilon_g \rho_g u_g(x, t) &= g_{g.in}(t) / F_{f.in}; \\ \frac{d}{dx} \varepsilon_d \rho_d u_g(x, t) &= g_{d.in}(t) / F_{f.in}; \\ \frac{d}{dx} \varepsilon_s \rho_s u_s(x, t) &= g_{s.in}(t) / F_{f.in}, \end{aligned} \quad (6)$$

where x_{in} – axial coordinate which corresponds to phase input, m; $g_{g.in}$, $g_{d.in}$ и $g_{s.in}$ – mass rates of gas, dust, material at the input point, kg/s; $F_{f.in}$ – area of inside kiln cross-section at the input point, m².

Mass conservation equations for chemicals

Continuity equation for phases (1)-(3) should be added by mass concentrations of chemical substances for expressing of chemicals balances.

Balance of gas compounds in the rotary kiln followed by mutual action of convection, of interface transfer between material and dust, between dust and gas, of intensity of chemical reactions in gas phase. Diffusion can be neglected taking into account low residence time for gas inside a kiln. Mass conservation equations for gas can be written in form:

$$\begin{aligned} \frac{\partial}{\partial t} [\varepsilon_g \rho_g \chi_{g(i)}] + \frac{\partial}{\partial x} [\varepsilon_g \rho_g u_g \chi_{g(i)}] = \\ = \sum_{j=1}^{rg} (S_{sg(i,j)} + S_{dg(i,j)}) + \sum_{j=1}^{rg} \text{sign}_{(i,j)} \cdot v_{g(i,j)} M_i R_{g(j)}, \end{aligned} \quad (7)$$

for the same reason diffusion term in the equation for dust was neglected:

$$\begin{aligned} \frac{\partial}{\partial t} (\varepsilon_d \rho_d \chi_{d(i)}) + \frac{\partial}{\partial x} [\varepsilon_d \rho_d u_g \chi_{d(i)}] = \\ = P_1 \chi_{s(i)} - P_2 \chi_{d(i)} + \sum_{j=1}^{rs} \text{sign}_{(i,j)} \cdot v_{s(i,j)} M_i R_{d(j)}, \end{aligned} \quad (8)$$

need in adding of diffusion term to the equation for material causes by experimental proved axial mixing of material followed by difference in axial velocity of big and small bulks:

$$\begin{aligned} \frac{\partial}{\partial t} (\varepsilon_s \rho_s \chi_{s(i)}) + \frac{\partial}{\partial x} [\varepsilon_s \rho_s u_s \chi_{s(i)}] - \frac{\partial}{\partial x} \left[\varepsilon_s \rho_s D_s \frac{\partial \chi_{s(i)}}{\partial x} \right] = \\ = - \left(P_1 \chi_{s(i)} - P_2 \chi_{d(i)} + \sum_{j=1}^{rs} v_{s(i,j)} M_i R_{s(j)} \right), \end{aligned} \quad (9)$$

where $\chi_{g(i)}$, $\chi_{d(i)}$ and $\chi_{s(i)}$ – mass parts of compound i in gas, dust and material; $S_{sg(i,j)}$ – intensity of mass transfer from material to gas for gas compound i after chemical reaction j in material, kg/(m³·s); $S_{dg(i,j)}$ – the same for dust-gas transfer, kg/(m³·s); $\text{sign}_{(i,j)}$ – if specie formed as a product of reaction j , then $\text{sign} = +1$, if specie destroyed in reaction j , then $\text{sign} = -1$; $v_{g(i,j)}$ and $v_{s(i,j)}$ – stoichiometric coefficient of gas compound i in reaction j taking place in gas or material phase; M_i – molar mass of compound i , kg/mol; $R_{g(j)}$, $R_{d(j)}$ and $R_{s(j)}$ – reaction flow of reaction j in gas, dust and material consequently, mol/(m³·s); rg – total number of gas reactions; D_s – coefficient of axial mixing (diffusion) for material, m²/s.

Initial conditions for equations (7)-(9) are given below (at $t = 0$):

$$\begin{aligned} \varepsilon_g \rho_g \chi_{g(i)}(x, t) = g_{g.in}(x) / [u_{g.in}(x) F_f]; \\ \varepsilon_d \rho_d \chi_{d(i)}(x, t) = g_{d.in}(x) / [u_{g.in}(x) F_f]; \\ \varepsilon_s \rho_s \chi_{s(i)}(x, t) = g_{s.in}(x) / [u_{s.in}(x) F_f]. \end{aligned} \quad (10)$$

Boundary conditions for equations (7)-(9) are given below (at $x = x_{in}$):

$$\begin{aligned} \varepsilon_g \rho_g \chi_{g(i)}(x, t) = \varepsilon_g \rho_g \chi_{g(i)}(t); \\ \varepsilon_d \rho_d \chi_{d(i)}(x, t) = \varepsilon_d \rho_d \chi_{d(i)}(t); \\ \varepsilon_s \rho_s \chi_{s(i)}(x, t) = \varepsilon_s \rho_s \chi_{s(i)}(t). \end{aligned} \quad (11)$$

Heat conservation equations

Heat balance of a rotary kiln depends on: change of enthalpy of gas flow, dust and material as a result of phase transformation and chemical reactions; heat transfer between phases and heat losses into atmosphere. We were suggesting that gas and dust temperatures are different. Thus heat conservation equation for gas phase can be written in form:

$$\begin{aligned} \frac{\partial}{\partial t} (\varepsilon_g \rho_g c_{pg} T_g) + \frac{\partial}{\partial x} [\varepsilon_g \rho_g u_g c_{pg} T_g] - \frac{\partial}{\partial x} \left[\varepsilon_g \lambda_g \frac{\partial T_g}{\partial x} \right] = \\ = \sum_{j=1}^{rg} \Delta H_{g(j)} R_{g(j)} + \sum_{j=1}^{rs} (\Delta H_{sg(i)} S_{sg(i,j)} + \\ + \Delta H_{dg(i)} S_{dg(i,j)}) + Q_{g.res}, \end{aligned} \quad (12)$$

for dust:

$$\begin{aligned} \frac{\partial}{\partial t} (\varepsilon_d \rho_d c_d T_d) + \frac{\partial}{\partial x} [\varepsilon_d \rho_d u_g c_d T_d] - \frac{\partial}{\partial x} \left[\varepsilon_d \lambda_d \frac{\partial T_d}{\partial x} \right] = \\ = \sum_{j=1}^{rs} \Delta H_{s(j)} R_{d(j)} - \sum_{j=1}^{rs} (\Delta H_{dg(i)} S_{dg(i,j)}) + Q_{d.res}, \end{aligned} \quad (13)$$

for material:

$$\begin{aligned} \frac{\partial}{\partial t} (\varepsilon_s \rho_s c_s T_s) + \frac{\partial}{\partial x} [\varepsilon_s \rho_s u_s c_s T_s] - \frac{\partial}{\partial x} \left[\varepsilon_s \lambda_s \frac{\partial T_s}{\partial x} \right] = \\ = \sum_{j=1}^{rs} \Delta H_{s(j)} R_{s(j)} - \sum_{j=1}^{rs} (\Delta H_{sg(i)} S_{sg(i,j)}) + Q_{s.res}, \end{aligned} \quad (14)$$

where λ – heat conductivity of gas, W/(m·K); c_{pg} – specific heat consumption of gas at constant pressure, J/(kg·K); c_s and c_d – specific heat consumption of material and dust, J/(kg·K); $Q_{g.res}$, $Q_{d.res}$ and $Q_{s.res}$ – resulted heat flow for gas, material and dust consequently, W/m³; $\Delta H_{g(j)}$ and $\Delta H_{s(j)}$ – heat of reaction j in gas or material, J/mol; $\Delta H_{sg(i)}$ and $\Delta H_{dg(i)}$ – change of enthalpy of compound i , which effervesce from material and dust into gas phase, J/kg.

Initial conditions for equations (12)-(14) (at $t = t_0$) can be expressed in form:

$$\begin{aligned} T_g(x, t_0) = T_{g0}(x); T_d(x, t_0) = T_{d0}(x); \\ T_s(x, t_0) = T_{s0}(x), \end{aligned} \quad (15)$$

where $T_0(x)$ – temperature of correspondent flow as a function of axial coordinate at the initial moment, K.

Boundary conditions for equations (12)-(14) (at $x = x_{in}$) can be expressed in form:

$$\begin{aligned} T_g(x_{in}, t) = T_{g.in}(t); T_d(x_{in}, t) = T_{d.in}(t); \\ T_s(x_{in}, t) = T_{s.in}(t). \end{aligned} \quad (16)$$

Boundary conditions for equations (12)-(14) (at $x = x_{out}$) satisfy convection condition:

$$\begin{aligned} -\varepsilon_g \lambda_g \frac{\partial T_g(x_{out}, t)}{\partial x} = 0; -\varepsilon_d \lambda_d \frac{\partial T_d(x_{out}, t)}{\partial x} = 0; \\ -\varepsilon_s \lambda_s \frac{\partial T_s(x_{out}, t)}{\partial x} = 0. \end{aligned} \quad (17)$$

Movement equation

Gas velocity changes as a function of axial coordinate, and its value defined by equation:

$$\frac{\partial}{\partial t} [\varepsilon_g \rho_g u_g] + \frac{\partial}{\partial x} \left[\varepsilon_g \rho_g \frac{u_g^2}{2} + \varepsilon_g p_g \right] = \frac{\partial}{\partial x} \left[\varepsilon_g \mu_g \frac{\partial u_g}{\partial x} \right]. \quad (18)$$

Material velocity was considered as constant.

Initial conditions for equation (18) (at $t = 0$):

$$u_{g(i)}(x, t) = u_{g,init}. \quad (19)$$

Boundary conditions for equation (18) (at $x = x_{in}$):

$$u_g(x, t) = u_g(t). \quad (20)$$

Dust and gas velocity are equal as it was described before.

Definition of equations coefficients

Definition of chemical reactions

Assume chemical reactions inside a flow are reversible and independent of other flows. In this case velocity of reaction j can be defined by formula:

$$R_j = k_{forw} \prod_{i=1}^{n_{j,forw}} c_{i,forw}^{v_{i,j}} - k_{back} \prod_{i=1}^{n_{j,back}} c_{i,back}^{v_{i,j}}, \quad (21)$$

where R_j – reaction flow as a result of reaction j , mol/(m³·s); k_{forw} and k_{back} – rate constant for forward and backward reactions, s⁻¹; c_{forw} and c_{back} – molar concentration of i reagents and products of j reaction, mol/m³; $n_{j,forw}$ and $n_{j,back}$ – number of reagents and products of j reaction.

Molar concentration of chemical compound in flow expressed by formula:

$$c_i = \chi_i \cdot \rho / M_i, \quad (22)$$

where ρ – density of reaction flow, kg/m³.

Rate constants for forward and backward reactions calculated by Arrhenius equation:

$$k = k_0 T^m \exp\left(-\frac{E_a}{R_{um} T}\right), \quad (23)$$

where k_0 – pre-exponential factor, s⁻¹; m – exponential factor at temperature; E_a – activation energy, J/mol; $R_{um} = 8,314$ – universal gas constant, J/(mol·K).

Equation (21) can be used for irreversible reactions with $k_{back} = 0$.

Heat of reaction j expressed by formula:

$$\Delta H_j = -\sum_{i=1}^{n_j} \text{sign}_{i,j} v_{i,j} h_i, \quad (24)$$

where h_i – molar enthalpy of compound i , J/mol.

Intensity of mass transfer between material and gas $S_{sg(i,j)}$ and between dust and gas $S_{dg(i,j)}$ calculated by formula:

$$S_{i,j} = v_{s(i,j)} M_i R_j, \quad (25)$$

which includes physic-chemical transformations inside solid phase with formation of gas compound.

Changing of enthalpy of gas compound i during transfer from material to gas $\Delta H_{sg(i)}$ and from dust to gas $\Delta H_{dg(i)}$ will be:

$$\Delta H_i = c_{pg}(T_g - T_x), \quad (26)$$

where x – index, transforms to s for transfer in system material-gas and transforms to d – for system dust-gas.

Definition of phases

Velocity of axial movement of material in the kiln depends on speed of kiln rotation and can be calculated by approximate formula:

$$u_s = 5,78 d_f \alpha_f \omega_f, \quad (27)$$

where α_f – kiln tilt, rad; ω_f – rotation rate of a kiln, rpm.

If we suggest that porosity of material is constant $\varepsilon_m = \text{const}$, then bulk density of material will depend only on real density of solid material:

$$\rho_s = \rho_{s0}(1 - \varepsilon_m),$$

where ρ_{s0} – real density of material, kg/m³.

Fraction of section of a kiln, loaded by material can be calculated with use of values for mass rate, bulk density and velocity of material by formula:

$$\varepsilon_s = (4m_{s1}) / (\rho_{s1} \pi d_f^2 u_s),$$

or

$$\varepsilon_s = 0,22 \frac{m_{s1}}{\rho_{s1} d_f^3 \alpha_f \omega_f}, \quad (28)$$

where m_{s1} – mass rate of material at the kiln inlet, kg/s; ρ_{s1} – bulk density of material at the kiln inlet, kg/m³; d_f – inside diameter of a kiln, m.

Given (20) and (27), equation (3) can be transformed to a form:

$$0,22 \frac{\partial}{\partial t} \left[\frac{m_{s1} \rho_s}{\rho_{s1} \omega_f} \right] + \frac{1,272}{d_f^2} \frac{m_{s1}}{\rho_{s1}} \frac{\partial \rho_s}{\partial x} = - \left[P_1 - P_2 + \sum_{i=1}^{ng} \sum_{j=1}^{rx} S_{sg(i,j)} \right]. \quad (29)$$

Other formula for calculation a fraction of section of a kiln loaded by material is shown below:

$$\varepsilon_s = [\Theta/2 - 2 \sin(\Theta/2) \cdot \cos(\Theta/2) / 4] / \pi, \quad (30)$$

where Θ – central angle of segment, loaded by material, rad.

Fraction of section occupied by dusty gas ($\varepsilon_g + \varepsilon_d$) can be calculated with formula (4).

Given known area of kiln section, fraction of this section loaded by material and dust we can calculate a fraction of gas area ε_g . After that we can calculate velocity of gas flow u_g taking into account mass rate of gas m_g , density ρ_g and fraction ε_g :

$$u_g = (4m_g) / (\rho_g \pi d_f^2 \varepsilon_g).$$

Maximum volume fraction of dust in gases can be approximated by empirical formula [2]:

$$\varepsilon_{d \max} = 2 \cdot 10^{-11} \varepsilon_g u_g^m \frac{d_f^{3/2}}{d_d \mathfrak{R}^3} \left(\frac{\omega_f}{\varepsilon_s} \right)^{2/3} \left(\frac{\rho_g}{\rho_d} \right)^{(1+m/3)}; \quad (31)$$

$$m = 0,25 \cdot d_{d \max} \mathfrak{R} / d_d,$$

where d_d and $d_{d \max}$ – average and maximum diameter of dust particles, m; \mathfrak{R} – size distribution parameter, defined in Rosin-Rammler equation ($\mathfrak{R} = 1,6$ for sand, $\mathfrak{R} = 1,7$ for sinter, $\mathfrak{R} = 0,68$ – magnesium ore, $\mathfrak{R} = 0,6$ – iron ore).

Definition of dust raising and precipitation

Mass transfer intensity between material and dust depends on intensity of raising P_1 and precipitation P_2 of dust.

If kiln operates in regime without dust injection then the dusty of gas flow at the hot side of a kiln (on the side of flame) depends on intensity of dust raising but at the middle and other side of a kiln it followed by value of maximum volume concentration of dust in the gas flow according to equation (31).

Results of experimental investigations [2] proved that maximum concentration of a dust in the gas flow achieved at the distance equal to 5-6 kiln diameters from inlet section. Experiment was occurring on a kiln which has no intensification devices for heat transfer and no gas discharge from material at the initial area. Thus, intensity of dust raising for this kiln area can be account by formula:

$$P_{1\text{ gen}} = 0,18 \frac{\rho_d u_g}{d_f} \varepsilon_{d\text{ max}} \quad (32)$$

Industrial investigations in VAMI (Saint-Petersburg, Russia) were carried out at the kilns with sizes 3,6×3,3×3,6×150 m at JSC «Pikalevo Alumina». Was demonstrated that intensity of raising is a function of kiln length [1, 2]. Around 55-65 % of total dust raising concentrates in chain zone, 25-30 % – in zone of carbonate dissociation and only 10 % of a dust arises in zones without inside devices for intensification for heat exchange and without discharging of gas compounds from material. Suggest that intensity of dust raising is constant inside selected zones we can describe this mechanism by piecewise function:

$$P_1 = k_{dp1} \cdot P_{1\text{ gen}} \quad \text{and} \quad k_{dp1} = \begin{cases} 3 & \text{if } S_{sg.co2} > 0; \\ 6 & \text{if } l \geq l_{chain}; \\ 1 & \text{if } S_{sg.co2} = 0 \text{ and } l < l_{chain}, \end{cases} \quad (33)$$

where l and l_{chain} – current axial coordinate of a kiln and chain length, m.

Zone of carbonates dissociation can be identified as a zone with CO_2 emission from material to gas by rule $S_{sg.co2} > 0$.

Accept that intensity of dust precipitation is equal to zero if maximum concentration $\varepsilon_{d\text{ max}}$ is not achieved and that it is equal to P_1 with opposite sign after achievement of $\varepsilon_{d\text{ max}}$. This brings us to next condition for kiln which operates without forced dust injection:

$$P_{2\text{ free}} = \begin{cases} 0 & \text{if } \varepsilon_d < \varepsilon_{d\text{ max}}; \\ -P_1 & \text{if } \varepsilon_d \geq \varepsilon_{d\text{ max}}. \end{cases} \quad (34)$$

Dust concentration increases higher then calculated by formula (31) in case of forced dust injection at the hot side of a kiln. Around 25% of injected dust precipitates at the sintering zone and immediately after it in the distance 0-60 m, around 40% precipitates in carbonization zone in distance 60-120 m, around 30% – in chain zone (120-180 m), rest of dust in quantity of 5% leave a kiln with gas flow [1].

If we suggest that intensity of dust precipitation is constant in marked zones, than mass exchange rate will be a piecewise function of kiln length:

$$P_{2\text{ forced}} = k_{dp2} \cdot \frac{g_{d.in}(t)}{F_{f.in} \cdot l_f} \quad \text{and} \quad k_{dp2} = \begin{cases} 0.40 & \text{if } S_{sg.co2} > 0; \\ 0.30 & \text{if } l \geq l_{chain}; \\ 0.25 & \text{if } S_{sg.co2} = 0 \text{ and } l < l_{chain}. \end{cases} \quad (35)$$

Then excess of a dust precipitates and dust concentration will be equal to maximum value $\varepsilon_{d\text{ max}}$ next condition must executes: $k_{dp2} = 0$.

Full intensity of dust precipitation is a sum of precipitation intensity for own and injected dust:

$$P_2 = P_{2\text{ free}} + P_{2\text{ forced}} \quad (36)$$

Volumetric fractions of gas and dust calculate with use of iteration scheme (4), (30)-(36).

Definition of mass transfer in drying zone

Evaporation of water to gas phase takes place in the drying zone. Drying process divides on two stages according to the standard representations. Mass transfer rate is constant at the first stage of drying and it depends only on saturated pressure of steam at the surface of material. Drying front penetrates in the deep of solid phase and drying rate start to decrease with the decreasing of humidity at the second stage. Intensity of drying at the second stage depends on relative humidity of material $\chi_{s,h2o}$ and coefficient of drying intensity. The position of interface between two drying stages depends on characteristics of solid material. For example, border point between two drying stages for cement slurry lies at humidity around $\chi_{s,h2o} = 5\%$, so drying rate follows by formula [3]:

$$S_{sg,h2o} = \begin{cases} S_{sg,h2o.const} & \text{if } \chi_{s,h2o} > 0,05; \\ S_{sg,h2o.fall} & \text{if } \chi_{s,h2o} \leq 0,05. \end{cases} \quad (37)$$

Next equation is right for period with constant drying intensity:

$$S_{sg,h2o.const} = \beta f_{vol} \rho_g (\chi_{sat,h2o} - \chi_{g,h2o}), \quad (38)$$

where β – mass transfer coefficient, m/s; f_{vol} – extent of development of evaporation surface, m^2/m^3 ; $\chi_{sat,h2o}$ and $\chi_{g,h2o}$ – gas humidity near material surface at saturation regime and actual humidity in gas flow consequently, kg/kg.

Humidity of saturation steam can be calculated by formula:

$$\chi_{sat,h2o} = (M_{h2o} \cdot p_{sat,h2o}) / (M_g \cdot p_g), \quad (39)$$

where $p_{sat,h2o}$ – saturation pressure for water at temperature T_g , Pa; M_{h2o} – molar mass of steam, kg/mol; M_g – molar mass of kiln gas, kg/mol.

For example, we can use next dependence between saturated pressure (Pa) and temperature (K) in diapason from 273,16 to 647,13 K with error less than 0,2 % [4]:

$$P_{sat,h2o} = \exp(73.649 - 7.2582 \cdot 10^3 / T_g - 7.3037 \ln(T_g) + 4.1653 \cdot 10^{-6} T_g^2), \quad (40)$$

Mass transfer in drying zone of a rotary kiln is closely connected with convective heat transfer. Next dependence between coefficients of mass transfer and convective heat transfer were developed in work [5]:

$$\beta = \sqrt{Le} \frac{\alpha_{gs,conv}}{\rho_g c_g} \left(\frac{P_g - \bar{p}_{g,h2o}}{P_g} \right), \quad (41)$$

where $Le = D_g/a_g$ – Lewis number; D_g – diffusion coefficient for water steam in gas phase, m^2/s ; $a_g = \lambda_g/(c_g \rho_g)$ – thermal diffusivity coefficient for gas, m^2/s ; $\alpha_{gs,conv}$ – coefficient of convective heat transfer in system gas-material, $W/(m^2 \cdot K)$; $\bar{p}_{g,h2o}$ – average steam pressure, Pa.

Difference between actual and average steam pressure can be expressed as:

$$P_g - \bar{p}_{g,h2o} = (P_{sat,h2o} - P_{g,h2o}) \cdot \ln \left(\frac{P_g - P_{g,h2o}}{P_g - P_{sat,h2o}} \right)^{-1}. \quad (42)$$

Next equation can be implemented for evaporation kinetics for period of falling drying intensity:

$$S_{sg,h2o,fall} = k_{dry} \chi_{s,h2o} \rho_s, \quad (43)$$

where k_{dry} – experimental drying coefficient, 1/s.

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ДИНАМИЧЕСКАЯ ОДНОМЕРНАЯ МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ВРАЩАЮЩЕЙСЯ ПЕЧИ. ЧАСТЬ 1: ОСНОВНЫЕ УРАВНЕНИЯ

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Аннотация: Рассмотрены общие вопросы математического описания трубчатых вращающихся печей с учетом протекающих в них сложных теплообменных процессов, химических превращений, изменения фазового состояния и кристаллической структуры обрабатываемого материала. Синтезирована динамическая одномерная математическая модель вращающейся печи, учитывающая теплофизику системы газ – материал – кладка – внешняя среда. Описан способ решения обратной задачи для математической модели вращающейся печи, заключающийся в оценке изменения параметров объекта по наблюдению за возмущениями сигналов на выходе из печи, перспективный для синтеза прогнозирующей адаптивной системы оптимального управления.

Ключевые слова: вращающаяся печь, математическая модель, уравнения сохранения, массоперенос.